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# ANALYSIS OF SOLID WASTE COMPOSITION. STATISTICAL TECHNIQUE TO DETERMINE SAMPLE SIZE

Dennis E. Carruth, et al

Bureau of Solid Waste Management
Cincinnati, Ohio

1969

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## Statistical Technique to Determine Sample Size

SW-19ts

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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service
Consumer Protection and Environmental Health Service
Environmental Control Administration
Bureau of Solid Waste Management
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I

ABSTRACT

This work analyzes data obtained after separating and weighing solid waste and presents a statistical technique to determine the minimum weight and number of samples needed to realistically and reliably estimate the characteristics of a given quantity of solid waste. Samples of solid waste of varying weights were taken and separated into nine components; each component was weighed, and the data were then statistically evaluated. It was determined that there is no significant variance among sample weight groups. The study recommends that 12, relatively small (200-lb) samples will validly reflect the composition of a given supply of solid waste. (Author)

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II

ABSTRACT

This work analyzes data obtained after separating and weighing solid waste and presents a statistical technique to determine the minimum weight and number of samples needed to realistically and reliably estimate the characteristics of a given quantity of solid waste. Samples of solid waste of varying weights were taken and separated into nine components; each component was weighed, and the data were then statistically evaluated. It was determined that there is no significant variance among sample weight groups. The study recommends that 12, relatively small (200-lb) samples will validly reflect the composition of a given supply of solid waste. (Author)

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# ANALYSIS OF SOLID WASTE COMPOSITION

## Statistical Technique to Determine Sample Size

Dennis E. Carruth and Albert J. Klee*

## INTRODUCTION

Whenever attempts are made to advance the frontiers of knowledge and to acquire additional understanding of a phenomenon, operation, or system, hypotheses must be formulated and tested. Certainly, the data gathered in the course of such studies must contribute to established objectives. Such a process of basic and applied research (reporting → description → explanation)[1] is universal to all creative disciplines and research endeavors.

The Bureau of Solid Waste Management is not unique in this respect nor is it a stranger to scientific research in attempting to broaden the existing horizons of knowledge in solid waste management. One objective is for Federal, State, county, and municipal governments, industry, and other interested and concerned institutions and individuals to obtain increased knowledge and better understanding of solid waste practices.

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Solid waste is defined by the Solid Waste Disposal Act of 1965 as:

> . . .garbage, refuse, and other discarded solid materials, including solid waste materials resulting from industrial, commercial, and agricultural operations, and from community activities, but does not include solids or dissolved material in domestic sewage or other significant pollutants in water resources, such as silt, dissolved or suspended solids in industrial waste water effluents, dissolved materials in irrigation return flows or other common water pollutants.[2]

To broaden the base of understanding in solid waste management, the Bureau's Division of Technical Operations is charged with the responsibility to provide technical assistance, to develop basic technical information on solid wastes and solid waste management practices, to manage the State planning grant program, and to apply management science to problems of solid waste handling and disposal. The commonly recognized methods of disposal, which can prove satisfactory, include sanitary landfilling, incinerating, and composting, as well as salvage and reuse operations.

## Solid Waste Composition Criteria

Through legislative action and increased public awareness, considerable interest and activity have recently developed in the solid waste field. Solid waste research, however, is primarily speculative in nature and is currently more concerned with gathering data and establishing hypotheses than with formulating broad generalizations or conclusions.

An important phase in data collection and survey design is quantifying and qualifying solid waste materials generated from a multitude of sources, including residential, commercial, industrial, and institutional origins. Yet, our knowledge of the physical and chemical composition of solid waste has remained undeveloped. Solid waste is not only complex in its composition, but also extremely variable because of such factors as geographical location, season of the year, social and economic conditions, and methods and frequency of collection.

For example, to design and operate a municipal incinerator and to evaluate its efficiency in processing solid waste, the Btu (heat) content of the fuel, i.e., solid waste, must be determined. Sanitary landfill, as a disposal method, requires a knowledge of solid waste composition to determine compaction properties, problems of leachates, gaseous emissions, and so forth. Composting requires the separation of glass, metals, rocks, and other nondegradable items from the otherwise compostible material.

To learn more about the nature of solid waste, a reliable and standard sampling procedure must be developed so that meaningful samples of solid waste can be obtained for analysis and study. Studies at Purdue University determined that analyzing a quarter-truckload of solid waste produced satisfactory data.[3] From this, the concept of quartering a given supply of solid waste before appropriate analysis and separation was developed. Further, a statistical analysis

gave the approximate number of sample households, in a homogeneous area of a community, needed to estimate such quantities as pounds per capita per day, pounds per household per day, cubic feet per capita per day, and bulk density. When solid wastes generated from a multitude of sources are analyzed, however, these concepts do not seem to apply.

## Sample Criteria

The preceding discussion indicates the need for quantifying and classifying solid waste materials. It is, of course, practically impossible and physically undesirable to separate, measure, weigh, and analyze all of the solid waste deposited at a particular facility. Therefore, should all the solid waste in a storage pit of an incinerator be separated and analyzed, or what portion thereof? In determining the physical composition of a truckload of solid waste, should the entire truckload be sampled, or rather a smaller fraction? What weight sample yields reasonable results and how many such samples are needed? Obtaining answers to these and similar questions is troublesome, yet important.

Samples must be taken in sufficient weight and number to realistically and reliably estimate the characteristics of a given quantity of solid waste.

> In the planning of a sample survey, a stage is always reached at which a decision must be made about the size of the sample. The decision is important. Too large a

> sample implies a waste of resources, and too small a sample diminishes the utility of the results. The decision cannot always be made satisfactorily, for often we do not possess enough information to be sure that our choice of sample size is the best one.[4]

Previous studies have not fully treated the problem of obtaining adequate samples of solid waste for analysis, nor have practices been developed to the extent of producing statistically comparable data. The variable composition of solid waste from a given source should be definable, however, and the following discussion, though specific in scope, should offer a general technique by which to analyze solid waste composition.

## DISCUSSION

### Incineration Studies

These studies, which are projects of considerable depth and scope, are undertaken by the Bureau's Division of Technical Operations upon request of appropriate State, local, and private concerns. Data are provided for comparative and operational purposes and, hopefully, for future recommendations.

The study utilizes a systems approach whereby the total process of incineration is observed, tested, and evaluated. Collection trucks and vehicles are weighed to provide true weights of the quantity of solid waste handled by the incinerator. Samples of solid waste material from the storage pit are taken, separated, and weighed to provide data on Btu content and composition of the input. Study of the incineration process also yields information on mechanical function, burning rate, burning temperature, pollution control systems, feed flow, normal and unique situations and conditions, and operating and control procedures. Samples of residue, gaseous and particulate emissions, and liquid effluents are taken to quantify and qualify the output of the systems and to ascertain the efficiency of the incineration process. Physical plant and disposal facilities, as well as economic and administrative support facilities, are also studied and evaluated.



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## Solid Waste Composition Classification

Determining the physical and quantitative composition of the input--solid waste--is a necessary step in studying the acceptability of incineration as a disposal method. The composition of the solid waste directly influences the Btu content of the waste, and determining the Btu content is necessary to evaluate the heat and combustion properties of this feed material.

After accounts of firsthand experiences and available literature were analyzed and reviewed, solid wastes were classified into nine categories. Each category is composed of materials similar in composition and Btu content; each category permits relatively direct comparisons with existing data; and each category is composed of materials that can be effectively separated, when the human element and reproducible methodology are considered.

The standard classification, based on nine mutually exclusive components, is (1) food waste; (2) garden waste; (3) paper products; (4) plastic, rubber, and leather; (5) textiles; (6) wood; (7) metals; (8) glass and ceramics; (9) ash, rocks, and dirt (inerts).

The classification is adaptable to analyze waste received at any disposal facility, although there are several significant differences among various methods of solid waste handling (e.g., whether or not "bulky" wastes, such as discarded refrigerators, are accepted).

## Method of Taking Samples

For 4 consecutive days, the study surveyed and collected data from an incineration facility that primarily handled residential and commercial solid wastes. A more thorough and complete study might conceivably evaluate the composition of solid waste based upon daily, weekly, monthly, seasonal, and yearly generation cycles.

Statistical analyses (in particular, Analysis of Variance) have shown that often there are significant differences in composition among solid waste samples taken on the same day and on different days during the week. This additional source of variability is, of course, unwanted in any experiment designed to compare precisions of samples of different weights. Unfortunately, securing a sufficient quantity of "homogeneous" material to perform such studies is impossible. Each truckload of material arriving at a disposal site represents a different population of waste. Indeed, much the same occurs with different grapple loads of waste taken from an incinerator pit even though attempts are usually made to mix the material. Thus, to reduce such sources of variation, samples of different weights must be taken at random times and days ("random block designs") or a systematic sampling plan ("partially random block designs") must be employed to achieve the same results. In this study, the latter course was followed so that samples of a particular weight class were partially randomized with regard to time and day. Those effects inflate the variances of the subsequent analyses, but they do not bias the variation of any weight class.

Past experience indicates that four men can reasonably separate and weigh approximately 2,000 lb of solid waste material in an average 8-hr day. The decision whether to analyze one 2,000-lb sample each day or whether analyzing smaller samples would be more reliable was resolved by taking approximately equal numbers of samples in three size ranges: Weight Group I, 1,400 to 1,700 lb; Weight Group II, 700 to 900 lb; and Weight Group III, 200 to 300 lb. During the study, a total of 11 such samples were taken--three in the largest size range and four each in the smaller ranges (Table 1).

The samples were taken with a front-end loader from the incoming waste storage pile. Particular care was taken to obtain samples similar to the bulk of the waste being delivered. To avoid biasing the samples by actually handpicking the material to be classified, however, each sample was removed in a single scoop from a point in the pile that appeared to contain a relatively complete mixture of the waste being delivered. Each sample scoop was subsequently placed on a cleaned area of the dumping floor and immediately hand-separated into the nine categories specified. The weight of each component was then recorded.

## Effect of Sample Weight on Precision

The study plan recognized the necessity of selecting samples of solid waste of varying weights. The amount of time and manpower available, of course, limited the number of samples taken. Of primary

TABLE 1

WEIGHT, IN POUNDS, OF COMPONENTS PER SAMPLE AND PER WEIGHT GROUP

| Component | Weight Group I | | | | Weight Group II | | | | | Weight Group III | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sample size | | | Total | Sample size | | | | Total | Sample size | | | | Total | Component totals |
| | 1,700 | 1,494.5 | 1,414 | | 906 | 867 | 749.5 | 700 | | 310 | 270 | 231 | 200 | | |
| Food waste | 248 | 63.0 | 156 | 467.0 | 88 | 67 | 26.5 | 38 | 219.5 | 35 | 6 | 13 | 12 | 66 | 752.5 |
| Garden waste | 166 | 60.0 | 16 | 242.0 | 31 | 55 | 15.5 | 2 | 103.5 | 4 | 6 | 5 | 9 | 24 | 369.5 |
| Paper products | 1,012 | 899.5 | 869 | 2,780.5 | 579 | 499 | 472.5 | 505 | 2,055.5 | 190 | 177 | 147 | 118 | 632 | 5,468.0 |
| Plastic, rubber, leather | 23 | 30.5 | 37 | 90.5 | 27 | 21 | 36.5 | 9 | 93.5 | 6 | 10 | 12 | 6 | 34 | 218.0 |
| Textiles | 32 | 27.0 | 13 | 72.0 | 21 | 14 | 21.5 | 17 | 73.5 | 2 | 3 | 7 | 15 | 27 | 172.5 |
| Wood | 18 | 27.5 | 21 | 66.5 | 27 | 5 | 26.5 | 52 | 110.5 | 8 | 3 | 2 | 6 | 19 | 196.0 |
| Metals | 99 | 150.5 | 104 | 353.5 | 73 | 93 | 64.0 | 48 | 278.0 | 34 | 43 | 20 | 12 | 109 | 740.0 |
| Glass and ceramics | 66 | 92.5 | 75 | 233.5 | 43 | 47 | 29.0 | 17 | 136.0 | 12 | 7 | 11 | 6 | 36 | 405.5 |
| Ash, rocks, dirt (inerts) | 36 | 144.0 | 123 | 303.0 | 17 | 66 | 57.5 | 12 | 152.5 | 19 | 15 | 14 | 16 | 64 | 519.5 |
| Group totals | | | | 4,608.5 | | | | | 3,222.5 | | | | | 1,011 | 8,842.0 |

importance, surely, is to determine the reliability and precision of selecting a specified number of samples of a certain weight.

To evaluate the original data, the weights of each component were converted to percentages of the total weight of the sample (Table 2). By grouping the data according to sample weight ranges, comparisons can be made among all samples, among samples within groups, and among groups. Such analysis leads to the ultimate objective of determining the optimum weight and number of samples to be taken. By presenting the data in percentage form, the percentage of a given component can be compared among samples and groups and among all components within a given sample or group. It is the variances of these percentages, however, that determine the sample weight and number of samples needed to adequately determine the composition of a given source of solid waste.

The data expressed as percentages follow a multinomial distribution. Before attempting statistical analysis, however, it is important to note that the individual percentages cannot, in general, be expected to follow a normal distribution. A normal distribution is considered a good approximation to the distribution of a percentage when the percentage lies between 30 and 70 percent. Only the paper component meets this criterion (Table 2); the other entries fall below the minimum acceptable 30 percent.

The following is an excellent normalizing transformation, however.

$$Y = 2 \arcsin \sqrt{X}$$

## TABLE 2

## PERCENTAGE (BY WEIGHT) OF COMPONENTS PER SAMPLE AND PER WEIGHT GROUP

| Components | Weight Group I | | | | Weight Group II | | | | | Weight Group III | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sample size (lb) | | | Avg. | Sample size (lb) | | | | Avg. | Sample size (lb) | | | | Avg. | Total avg. |
| | 1,700 | 1,494.5 | 1,414 | | 906 | 867 | 749.5 | 700 | | 310 | 270 | 231 | 200 | | |
| Food waste | 14.58 | 4.21 | 11.03 | 10.13 | 9.71 | 7.72 | 3.53 | 5.42 | 6.81 | 11.29 | 2.22 | 5.62 | 6.00 | 6.53 | 8.51 |
| Garden waste | 9.76 | 4.01 | 1.13 | 5.25 | 3.42 | 6.34 | 2.06 | 0.28 | 3.21 | 1.29 | 2.22 | 2.16 | 4.50 | 2.37 | 4.17 |
| Paper products | 59.52 | 60.18 | 61.45 | 60.33 | 63.90 | 57.55 | 63.04 | 72.14 | 63.79 | 61.29 | 65.55 | 63.63 | 59.00 | 62.51 | 61.84 |
| Plastic, rubber, leather | 1.35 | 2.04 | 2.61 | 1.96 | 2.98 | 2.42 | 4.86 | 1.28 | 2.90 | 1.93 | 3.70 | 5.19 | 3.00 | 3.36 | 2.46 |
| Textiles | 1.88 | 1.80 | 0.91 | 1.56 | 2.31 | 1.61 | 2.86 | 2.42 | 2.25 | 0.64 | 1.11 | 3.03 | 7.50 | 2.67 | 1.95 |
| Wood | 1.05 | 1.84 | 1.48 | 1.44 | 2.98 | 0.57 | 3.53 | 7.42 | 3.13 | 2.58 | 1.11 | 0.86 | 3.00 | 1.88 | 2.21 |
| Metals | 5.82 | 10.07 | 7.35 | 7.67 | 8.05 | 10.72 | 8.53 | 6.85 | 8.63 | 10.96 | 15.92 | 8.65 | 6.00 | 10.78 | 8.37 |
| Glass and ceramics | 3.88 | 6.18 | 5.30 | 5.07 | 4.74 | 5.42 | 3.86 | 2.42 | 4.22 | 3.87 | 2.59 | 4.76 | 3.00 | 3.56 | 4.58 |
| Ash, rocks, dirt (inerts) | 2.11 | 9.63 | 8.69 | 6.57 | 1.87 | 7.61 | 7.67 | 1.71 | 4.73 | 6.12 | 5.55 | 6.06 | 8.00 | 6.33 | 5.87 |
| Totals (percent) | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± | 100± |

where X is the original percentage value of a component expressed as a decimal,

and Y is the transformed value of X.

After the data of Table 2 are similarly transformed and then the sum of squares about the mean (SSM) is calculated for each component within each weight group, the data in Table 3 are obtained.

A commonly used measurement of relative precision is the coefficient of variation, which is defined as the standard deviation (square root of the variance) of a group of measurements divided by its mean. The result is then usually multiplied by 100 to convert to percent.

$$CV = (\sqrt{V} \; / \; \bar{Y}) \; 100^{*}$$

To facilitate visual examination, the coefficients of variation are summarized separately (Table 4).

An examination of the coefficients of variation of the three groups indicates no discernible pattern. For example, precision does not appear to increase as the sample weight is increased. The food waste and garden waste components have relatively high coefficients of variation but that of the paper products component is low. Irregularly high coefficients include textiles in Group III and wood in Group II, whereas the coefficients of variation of the inerts component vary erratically.

The Cochran C test[5] may be employed to determine, in a rigorous manner, (the Coefficient of Variation Table only provides insight) whether the variances among sample weight groups can be pooled for a

*See Table 3 for definition of terms.

TABLE 3

CRITICAL STATISTICS* OBTAINED FROM TABLE 2

(AFTER NORMALIZING TRANSFORMATION)

| Component | Weight Group I | | | | | Weight Group II | | | | | Weight Group III | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{Y}$ | SSM | DF | V | CV | $\bar{Y}$ | SSM | DF | V | CV | $\bar{Y}$ | SSM | DF | V | CV |
| Food waste | 0.6281 | 0.0766 | 2 | 0.0383 | 31.2 | 0.5111 | 0.0431 | 3 | 0.0144 | 23.5 | 0.4909 | 0.0715 | 3 | 0.0238 | 31.4 |
| Garden waste | 0.4074 | 0.0796 | 2 | 0.0398 | 48.9 | 0.3165 | 0.0782 | 3 | 0.0261 | 51.0 | 0.3129 | 0.0205 | 3 | 0.0068 | 26.3 |
| Paper products | 1.7790 | 0.0001 | 2 | 0.0001 | 0.5 | 1.8615 | 0.0447 | 3 | 0.0149 | 6.6 | 1.8238 | 0.0124 | 3 | 0.0041 | 3.5 |
| Plastic, rubber, leather | 0.2816 | 0.0037 | 2 | 0.0019 | 15.4 | 0.3335 | 0.0243 | 3 | 0.0081 | 27.0 | 0.3657 | 0.0159 | 3 | 0.0053 | 19.9 |
| Textiles | 0.2452 | 0.0046 | 2 | 0.0023 | 19.5 | 0.3029 | 0.0039 | 3 | 0.0013 | 11.9 | 0.3217 | 0.1042 | 3 | 0.0347 | 57.9 |
| Wood | 0.2383 | 0.0024 | 2 | 0.0012 | 14.5 | 0.5565 | 0.5571 | 3 | 0.1857 | 77.4 | 0.2680 | 0.0189 | 3 | 0.0063 | 29.6 |
| Metals | 0.5579 | 0.0117 | 2 | 0.0059 | 13.8 | 0.5986 | 0.0106 | 3 | 0.0035 | 9.9 | 0.6508 | 0.0562 | 3 | 0.0187 | 21.0 |
| Glass and ceramics | 0.4478 | 0.0047 | 2 | 0.0024 | 10.9 | 0.3992 | 0.0117 | 3 | 0.0039 | 15.6 | 0.3778 | 0.0081 | 3 | 0.0027 | 13.7 |
| Ash, rocks, dirt (inerts) | 0.5146 | 0.0755 | 2 | 0.0378 | 37.8 | 0.4212 | 0.0928 | 3 | 0.0309 | 189.3 | 0.5145 | 0.0046 | 3 | 0.0015 | 7.5 |

*KEY: $\bar{Y}$ is the mean of n observations (transformed) for a component within a group; $\bar{Y} = \Sigma X_i/n$.
SSM is the summation of the squares about the mean; $SSM = \Sigma(Y_i - \bar{Y})^2$.
DF is the degrees of freedom; $DF = n - 1$.
V is the variance; $V = SSM/DF$ ($\sqrt{V}$ is the standard deviation).
CV is the coefficient of variation; $CV = (\sqrt{V}/\bar{Y})\ 100$.

TABLE 4

COEFFICIENTS OF VARIATION

(EXPRESSED AT PERCENTAGES)

| Components | GROUP I | GROUP II | GROUP III |
|---|---|---|---|
| Food waste | 31.2 | 23.5 | 31.4 |
| Garden waste | 48.9 | 51.0 | 26.3 |
| Paper products | 0.5 | 6.6 | 3.5 |
| Plastic, rubber, leather | 15.4 | 27.0 | 19.9 |
| Textiles | 19.5 | 11.9 | 57.9 |
| Wood | 14.5 | 77.4 | 29.6 |
| Metals | 13.8 | 9.9 | 21.0 |
| Glass and ceramics | 10.9 | 15.6 | 13.7 |
| Ash, rocks, dirt (inerts) | 37.8 | 189.3 | 7.5 |

given component. The test compares the sum of a group of variances with the largest member of the group. Results of the test indicate that, at the 5 percent risk level, pooling of variances across groups is indeed justified provided the "questionable deviates," represented by the textile component in Group III and the wood component in Group II, are removed.

Additional use of the Cochran C test indicates that pooling variances among components is also justifiable. Thus, a single pooled estimate of variance, which yields an estimated standard deviation, s, for all transformed observations of 0.1413 at 72 degrees of freedom,* is obtained from Table 3.

The conclusion to this portion of the analysis is that there is no difference in precision among the three groups. Accordingly, a "small" (200-lb) sample is as adequate as a larger one. The desirability of taking small samples, supported by the preceding statistical analysis, is reinforced by the fact that they are much easier to handle. Past experience has shown that separating "small" samples is simpler and more accurate from a human standpoint than separating "large" ones.

## Statistical Technique to Determine Sample Size

To determine the number of samples required for composition analysis, the following normal approximation is adequate.

*As percentages, there are only eight independent observations per sample because the ninth component is predictable after the percent values for the first eight have been determined. The transformed values, however, are not subject to such predictability and thus yield nine independent observations per sample. All components from the samples in the three groups are included in this estimate, as this best reflects conditions to be found in practice during a 4-day sampling period.

$$n = \left(\frac{z\ s}{\delta}\right)^2$$

where z is the standard normal deviate for the confidence level desired,

s is the estimated standard deviation (transformed basis),

$\delta$ is the sensitivity (transformed basis), defined below,

and n is the number of samples required.

At 90 percent confidence, z equals 1.645; s equals 0.1413, as just noted. The sensitivity, transformed basis, is determined as follows: transform X and either $X + \Delta$ or $X - \Delta$ by the previously mentioned arcsin transformation (specific values shown in Table 5), where X is the expected percentage of the component in question and $\Delta$ is the desired precision for the percentage to be estimated (see example for further clarification of $\Delta$). The choice of sign for $X \pm \Delta$ is positive if X is less than 0.50 and negative if X is greater than 0.50. Calculate the sensitivity,

$$\delta = \left| 2 \arcsin \sqrt{X} - 2 \arcsin \sqrt{X \pm \Delta} \right|$$

and compute n (number of samples required) by the formula given.

Example. Suppose one wishes to estimate the percentage of metals in raw solid waste to within two (2) percentage units with a confidence of 90 percent. Metals are expected to be found at a concentration of approximately 9 percent. The first step is to convert the average

TABLE 5

ARCSIN TRANSFORMATION

$Y = 2 \arcsin \sqrt{X}$

| X | Y | X | Y | X | Y | X | Y | X | Y |
|---|---|---|---|---|---|---|---|---|---|
| .001 | .0633 | .041 | .4078 | .36 | 1.2870 | .76 | 2.1177 | .971 | 2.7993 |
| .002 | .0895 | .042 | .4128 | .37 | 1.3078 | .77 | 2.1412 | .972 | 2.8053 |
| .003 | .1096 | .043 | .4178 | .38 | 1.3284 | .78 | 2.1652 | .973 | 2.8115 |
| .004 | .1266 | .044 | .4227 | .39 | 1.3490 | .79 | 2.1895 | .974 | 2.8177 |
| .005 | .1415 | .045 | .4275 | .40 | 1.3694 | .80 | 2.2143 | .975 | 2.8240 |
| .006 | .1551 | .046 | .4323 | .41 | 1.3898 | .81 | 2.2395 | .976 | 2.8305 |
| .007 | .1675 | .047 | .4371 | .42 | 1.4101 | .82 | 2.2653 | .977 | 2.8371 |
| .008 | .1791 | .048 | .4418 | .43 | 1.4303 | .83 | 2.2916 | .978 | 2.8438 |
| .009 | .1900 | .049 | .4464 | .44 | 1.4505 | .84 | 2.3186 | .979 | 2.8507 |
| .010 | .2003 | .050 | .4510 | .45 | 1.4706 | .85 | 2.3462 | .980 | 2.8578 |
| .011 | .2101 | .06 | .4949 | .46 | 1.4907 | .86 | 2.3746 | .981 | 2.8650 |
| .012 | .2195 | .07 | .5355 | .47 | 1.5108 | .87 | 2.4039 | .982 | 2.8725 |
| .013 | .2285 | .08 | .5735 | .48 | 1.5308 | .88 | 2.4341 | .983 | 2.8801 |
| .014 | .2372 | .09 | .6094 | .49 | 1.5508 | .89 | 2.4655 | .984 | 2.8879 |
| .015 | .2456 | .10 | .6435 | .50 | 1.5708 | .90 | 2.4981 | .985 | 2.8960 |
| .016 | .2537 | .11 | .6761 | .51 | 1.5908 | .91 | 2.5322 | .986 | 2.9044 |
| .017 | .2615 | .12 | .7075 | .52 | 1.6108 | .92 | 2.5681 | .987 | 2.9131 |
| .018 | .2691 | .13 | .7377 | .53 | 1.6308 | .93 | 2.6062 | .988 | 2.9221 |
| .019 | .2766 | .14 | .7670 | .54 | 1.6509 | .94 | 2.6467 | .989 | 2.9315 |
| .020 | .2838 | .15 | .7954 | .55 | 1.6710 | .95 | 2.6906 | .990 | 2.9413 |
| .021 | .2909 | .16 | .8230 | .56 | 1.6911 | .951 | 2.6952 | .991 | 2.9516 |
| .022 | .2978 | .17 | .8500 | .57 | 1.7113 | .952 | 2.6998 | .992 | 2.9625 |
| .023 | .3045 | .18 | .8763 | .58 | 1.7315 | .953 | 2.7045 | .993 | 2.9741 |
| .024 | .3111 | .19 | .9021 | .59 | 1.7518 | .954 | 2.7093 | .994 | 2.9865 |
| .025 | .3176 | .20 | .9273 | .60 | 1.7722 | .955 | 2.7141 | .995 | 3.0001 |
| .026 | .3239 | .21 | .9521 | .61 | 1.7926 | .956 | 2.7189 | .996 | 3.0150 |
| .027 | .3301 | .22 | .9764 | .62 | 1.8132 | .957 | 2.7238 | .997 | 3.0320 |
| .028 | .3363 | .23 | 1.0004 | .63 | 1.8338 | .958 | 2.7288 | .998 | 3.0521 |
| .029 | .3423 | .24 | 1.0239 | .64 | 1.8546 | .959 | 2.7338 | .999 | 3.0783 |
| .030 | .3482 | .25 | 1.0472 | .65 | 1.8755 | .960 | 2.7389 | | |
| .031 | .3540 | .26 | 1.0701 | .66 | 1.8965 | .961 | 2.7440 | | |
| .032 | .3597 | .27 | 1.0928 | .67 | 1.9177 | .962 | 2.7492 | | |
| .033 | .3654 | .28 | 1.1152 | .68 | 1.9391 | .963 | 2.7545 | | |
| .034 | .3709 | .29 | 1.1374 | .69 | 1.9606 | .964 | 2.7598 | | |
| .035 | .3764 | .30 | 1.1593 | .70 | 1.9823 | .965 | 2.7652 | | |
| .036 | .3818 | .31 | 1.1810 | .71 | 2.0042 | .966 | 2.7707 | | |
| .037 | .3871 | .32 | 1.2025 | .72 | 2.0264 | .967 | 2.7762 | | |
| .038 | .3924 | .33 | 1.2239 | .73 | 2.0488 | .968 | 2.7819 | | |
| .039 | .3976 | .34 | 1.2451 | .74 | 2.0715 | .969 | 2.7876 | | |
| .040 | .4027 | .35 | 1.2661 | .75 | 2.0944 | .970 | 2.7934 | | |

percentage and the acceptable precision to decimals, i.e., 0.09 and 0.02, respectively. The corresponding values of Y (the transformed quantities) obtained from Table 5 (X equals 0.09 and X + $\Delta$ equals 0.11, since X is less than 0.50) are 0.6094 and 0.6761. Thus, $\delta$ = 0.0667, and

$$n = \left( \frac{(1.645)\ (0.1413)}{0.0667} \right)^2 = 12.14$$

or n is approximately equal to 12 samples.

By performing a similar computation for each of the nine components and by using the expected percentages based upon the averages of the Group III samples, the following number of samples required for each component are obtained (assuming 90 percent confidence and a reasonable error of no more than 2 percent).

| Component | Number of samples size |
|---|---|
| 1. Food waste | 9 |
| 2. Garden waste | 6 |
| 3. Paper products | 20 |
| 4. Plastic, rubber, leather | 8 |
| 5. Textiles | 10 |
| 6. Wood | 10 |
| 7. Metals | 11 |
| 8. Glass and ceramics | 11 |
| 9. Ash, rocks, dirt (inerts) | 12 |

If all components are considered equally important to the total analysis and if a conservative approach is taken, a maximum number of 20 samples is needed to analyze the composition of solid waste. With the exception of the paper component, however, 12 samples should prove

adequate.* Further, within the confines and limitations of the study, the preceding analysis suggests that 12 of the smaller (200-lb) samples will yield valid and reliable results for composition evaluation.

*This conclusion is, of course, based upon an assumption that at 90 percent confidence, reasonable error should be limited to no more than 2 percentage units. If this criterion were relaxed, the necessary sample size would be reduced accordingly.

## SUMMARY AND CONCLUSIONS

Important to the study of problems associated with solid waste management is the definition, analysis, and evaluation of the generated waste. Such materials entering a solid waste management stream, require storage, collection, handling and processing, and ultimate disposal or salvage and reuse. Developing a reliable and standard sampling procedure to determine the composition of solid waste materials generated from any of a multitude of sources is of primary importance.

It is undesirable and impracticable to separate, measure, weigh, and analyze all the solid waste generated at specified sources or disposed of at particular facilities. Thus, to balance the resources available with the need for accurate results, appropriate statistical analysis holds promise of yielding information regarding optimum weights and number of samples to be taken. By initiating a planned sample survey, the composition of solid waste can be determined and evaluated within acceptable statistical limits.

To draw conclusions and to make recommendations based upon the preceding analysis, the study analyzed one incinerator operation and, more specifically, its solid waste composition for a 4-day period of time. A more thorough and complete analysis would also consider monthly, seasonal, yearly, and cyclical variations. Note also that *solid waste must be properly mixed* to yield adequate samples from a given supply



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of waste materials; and further, that if the criteria to classify solid waste differ from those applied in selecting the nine categories used in this analysis, the results may be influenced significantly.

With these facts in mind, the study of this incinerator indicates that there is no significant variance among sample weight groups. Consequently, smaller (200-lb) samples will yield valid results in determining the composition of a given supply of solid waste. Data from later studies, which are presently being evaluated and analyzed, appear to confirm and verify the results of this analysis. Indeed, it may be found that samples somewhat smaller than 200 lb will provide acceptable results.

It is encouraging to note that, upon the basis of this study, smaller samples than previously thought adequate hold promise of providing reasonable and reliable information concerning the composition of solid waste.

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